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Supereflexive Banach spaces

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FOURTH WINTER SCHOOL (1976).

SUPERREFLEXIVE BANACH SPACES

by

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A Banach space X mimics a Banach space Y if for each finite dimensional subspace $L \subset Y$ and $\varepsilon > 0$, there is a linear operator $T: L \rightarrow X$ with $\|T\| \cdot \|T^{-1}\| \leq 1 + \varepsilon$

Examples: 1) $c_0(N)$ mimics any Banach space (this is easy to prove)

2) A. Dvoretzky: Any Banach space mimics Hilbert space

3) J. Lindenstrauss, H. Rosenthal: Any Banach space X mimics its bidual X^{**} - so called local reflexivity of any Banach space -

A norm of a Banach space X is uniformly rotund if for each

$$\varepsilon > 0, \inf_{\substack{\|x\|=\|y\|=1 \\ \|x-y\| \geq \varepsilon}} (1 - \left| \frac{x+y}{2} \right|) > 0$$

A set $\{x_1, \dots, x_{2n+1}\}$ of a Banach space X is an $(n - \varepsilon)$ tree in X if

$$x_j = \frac{1}{2} \cdot (x_{2j} + x_{2j+1}), \|x_{2j} - x_{2j+1}\| \geq \varepsilon, j=1, 2, \dots, n.$$

A proof of the following theorem of
was discussed:

Theorem (R.C. James, P. Enflo). The following properties of a Banach space X are equivalent:

a) X mimics only reflexive Banach spaces (i.e. X is so called superreflexive)

- b) X admits an equivalent uniformly rotund norm
- c) X admits an equivalent uniformly Fréchet smooth norm
- d) for each $\epsilon > 0$, there is an integer n such that no $(n - \epsilon)$ tree lies in the unit ball of X

References: The works of R.C. James and P. Enflo - see e.g. the last edition of Day's book on Normed spaces.