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Borel and weakly Borel sets in Banach spaces

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Borel and weakly Borel sets in Banach spaces

by D.Preiss, Prague

The following interesting theorem was proved by Edgar.

Theorem: If a Banach space B admits an equivalent locally uniformly rotund norm then weak and strong Borel sets in B coincide.

Since it does not seem to be generally known it may be worthwhile to sketch here its proof. For $A \subset B$ define $\alpha(A) = \inf \{ \varepsilon > 0 ; x \in A \Rightarrow B(x, \varepsilon) \cap A \text{ is a weak neighbourhood of } x \text{ in } A \}$. If, for any natural n , the space B can be covered by countably many weakly Borel sets A_i^n with $\alpha(A_i^n) < \frac{1}{n}$ then any norm closed set F equals $\bigcap_{n=1}^{\infty} \bigcup_{i=1}^{\infty} \overline{A_i^n \cap (F \cap A_i^n)^w}$.

Suppose now that B has a LUR norm; it implies that on any sphere weak and strong topology coincide. Let $S(u, \nu) = \{x; u < \|x\| \leq \nu\}$ (u, ν rational numbers) and let $S^\varepsilon(u, \nu)$ be the union of all weakly open subsets of $S(u, \nu)$ with diameter less than ε . Since $\alpha(S^\varepsilon(u, \nu)) < \varepsilon$, it is sufficient to prove that for each $\varepsilon > 0$ the sets $S^\varepsilon(u, \nu)$ cover B . If $x \in B$ then $B(x, \frac{\varepsilon}{2}) \supset \{y; \|y\| = \|x\|, | \langle x'_1, (y-x) \rangle | < \delta \}$ for some $x'_1 \in B'$, $\|x'_1\| = 1$, $\delta > 0$. Choosing $u < \|x\| < \nu$ sufficiently close to $\|x\|$ we find that $B(x, \frac{\varepsilon}{2})$ is a weak neighbourhood of x in $S(u, \nu)$, thus $x \in S^\varepsilon(u, \nu)$.

Let us remark that B admits a LUR norm if it is weakly compactly generated or, more generally, weakly analytic (as was recently shown by Vařák). Let us also remark that recently Talagrand proved that in ℓ^∞ weakly and strongly Borel sets do not coincide.