

Gerhard G. Thomas

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## On Permutographs

Gerhard G. Thomas

**Summary:** The mathematical analysis of many-valued logic and of its supplement, the logic of value-contextures\*, leads to structures of permutations and problems of combinatorics. This exploration uses methods of graph theory. The fundamental operator of logic – the (many-valued) negation – was picked out to demonstrate something about the connection of many-valuedness and contextures. The network of negations of this logic are *permutographs*: regular graphs on permutations. Homogeneousess of knots (points) in connected simple graphs will introduced.

### Introduction

The following graph theoretical structures and problems emerged in the context of questions of the many-valued logic of *Gotthard Günther*. In my context the focal point is put on the presentation of mathematical problems, only slight attention is given to many-valued logic, i.e. there will be only some short (technical) annotations concerning the field of many-valued logic. In many-valued logic – including also the case of  $n = 2$  (that means the classic mathematical logic respectively the Aristotelian logic or predicative logic) – the focus is on  $n$  logical values. These values may not necessarily be truth values (Wahrheitswerte).

In the approach of *Günther* these logical values are homogeneous and there exists no relation of subsumption or ordination between them and they are for the present independent.

$\boxed{1}$  ,  $\boxed{2}$  ,  $\boxed{3}$  , ... ,  $\boxed{n}$  .

These values can be connected by logical operators. As we shall see soon the attributes of the logical values together with the negation operator  $N$  (negator) can be described by structures of permutations.

A *negator*  $N$  is a one-placed function, defined on two values  $x_1, x_2$

$$N(x_1) = x_2 \text{ and } N(x_2) = x_1.$$

The logical variables  $p, q, r, \dots$  are defined on the range of logical values

$1$  ,  $2$  ,  $3$  , ... ,  $n$  .

For  $n = 2$  we get the well-known figure

p	N(p)
1	2
2	1

If  $n = 3$ , then we get already five different negations of  $p$ .

---

\*For detailed introduction into the philosophy of many-valued logic and contextures see [3], especially volume 3. Articles are written in english or german.

p	$N_1(p)$	$N_2(p)$	$N_1N_2(p)$	$N_2N_1(p)$	$N_1N_2N_1(p)$
1	2	1	2	3	3
2	1	3	3	1	2
3	3	2	1	2	1

p and its five negations correspond exactly to 3! permutations of degree 3. Also for  $n = 3$  holds  $N_1N_1(p) = p$ .

If we transform logical negation problems into graph theoretical structures, we will see that exactly  $n - 1$  negators are sufficient to generate all  $n!$  states of negation (including the identity).

#### Value-contextures generate permutographs

Be  $\Pi$  the set of  $n!$  permutations. The elements of  $\Pi$  correspond to the  $n!$  possible states of a  $n$ -valued logic of negations.

The Negator  $N$  shall be an one-place relation on to values  $x_1, x_2$

$$x_1 \xleftrightarrow{N} x_2$$

$N(x_1) = x_2$  and  $N(x_2) = x_1$  hold at the same time.

In the two-valued logic  $x_1$  and  $x_2$  stand for 'true' and 'false'. In combinatorics we call the exchange of two integers (or elements) *transposition*. When both the integers of a transposition  $t$  are in ascending order ( $i, i+1$ ), then we name  $t$  *standard transposition*.

There are  $\binom{n}{2}$  different transpositions. They form the set  $TR$  of transpositions. If we apply a qualified sequence of transpositions

$$t_{i_1}, t_{i_2}, t_{i_3}, \dots, t_{i_r}$$

to a permutation  $\pi$ , so we get all of the other  $n! - 1$  permutations after a finite number of steps. In other words: From every permutation  $\pi$  you can construct the whole set  $\Pi$  of permutations by using the right transpositions. The following theorem says, that it is not necessary to use all  $\binom{n}{2}$  transpositions for the construction of  $\Pi$ .

A connected graph  $T$  is called *tree*, if the graph is connected and acyclic. A tree of  $n$  knots (points) has  $n-1$  edges (lines).

**THEOREM:** Be  $X := \{1, 2, \dots, n\}$ . A set  $T$  of  $n-1$  transpositions generates the symmetric group  $S_n$  (with  $n!$  elements) if, and only if, the graph  $(X, T)$  forms a tree.

The proof of this theorem you will find, for example in *Berge* [1].

Be (1)  $X := \{1, 2, \dots, n\}$  the set of  $n$  values;

(2)  $\Pi := (\pi_1, \pi_2, \dots, \pi_{P(r,n)})$  the set of permutations;  $P(r,n) :=$  total number of permutations;

(3)  $T := (t_1, t_2, \dots, t_m)$   $2 \leq m \leq \binom{n}{2}$  a set of transpositions of elements of  $X$ ;

(4) the *contexture*  $CT = (X, T)$  is a (connected) graph with  $|X|$  knots and  $|T|$  edges;

(5)  $\Pi$  is interpreted as the set of knots of graph  $PG$ ;

(6) two knots  $\pi_i, \pi_j \in \Pi$  are only then connected, if there exists a  $t \in T$ , which transforms

$$\pi_i \xleftrightarrow{t} \pi_j;$$

(7) all  $t \in T$  are edges of the  $|\Pi|$  knots of a graph  $PG$ ;

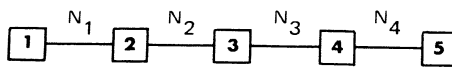
if (1) to (7) holds then  $PG = PG(\Pi, CT)$  is called a *permutograph*.

**Remark:** Every permutograph  $PG$  is  $m$ -regular.

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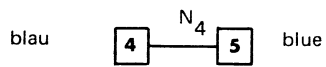
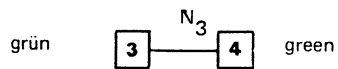
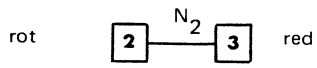
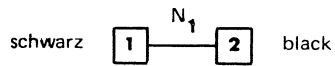
**On Permutographs**

Erzeugungsbaum

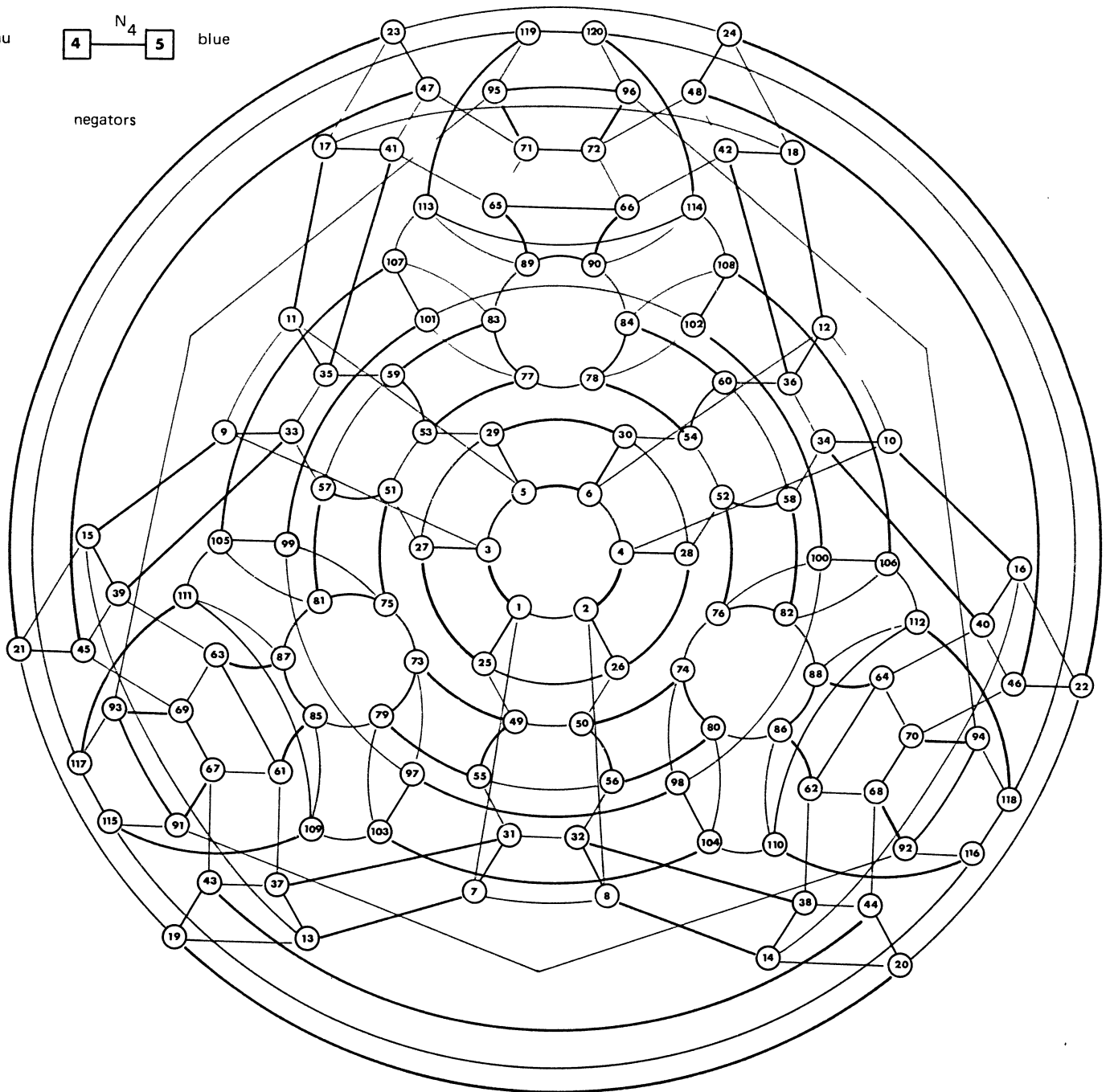


generating tree

Negatoren

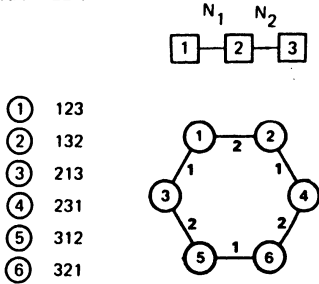


negators

PG<sub>1</sub> ( 5 ) : Mandala of Negations

Because theorem 1 holds, it is clear that  $n-1$  transpositions are enough to connect all  $\pi \in \Pi$ . That means the minimal contexture CT is a tree. In this case the graph PG is  $(n-1)$ -regular.

## EXAMPLE 1



tree-contexture of values 1,2,3 forms a *line*.

Negator  $N_1$  changes  $1 \leftrightarrow 2$

Negator  $N_2$  changes  $2 \leftrightarrow 3$

The tree-contexture describes the generating scheme of permutographs.

These sequences of negations form the identity:

$$N_1 N_2 N_1 N_2 N_1 N_2 \pi = \pi$$

$$N_2 N_1 N_2 N_1 N_2 N_1 \pi = \pi$$

Permutograph PG(  $\{3!\}, \{1,2,3\}$  )

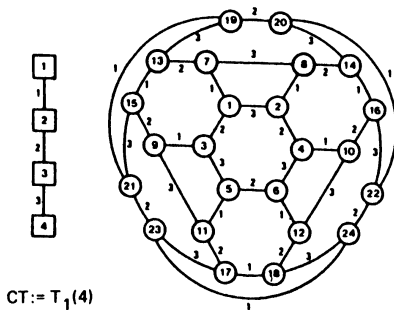
In *Harary* [5] you find diagrams of unlabelled trees for  $n = 1, 2, \dots, 10$ . The following table is an extraction of *Sloane* [7].

n	3	4	5	6	7	8	9	10	11	12	13	14
T(n)	1	2	3	6	11	23	47	106	235	551	1301	3159

number of different unlabelled trees

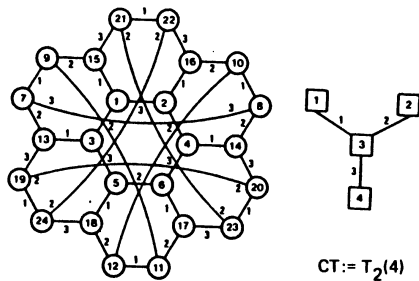
*Cayley* [2] found in 1889 that there exist  $n^{n-2}$  labelled trees. That means, for a given  $n$  there exist  $n^{n-2}$  different negation systems with a minimum number of negators (if the arrangement of the values is also regarded). Since all labelled trees generate isomorphic permutographs, it is sufficient to regard only *one* value contexture.

For  $n = 4$  there exist 2 different tree-contextures and therefor 2 different permutographs on 4-permutations:



CT :=  $T_1(4)$

PG(  $4!$ ,  $T_1(4)$  )



CT :=  $T_2(4)$

PG(  $4!$ ,  $T_2(4)$  )

The coloured figure on the folded page is one possible geometrical representation of the permutograph PG( $5!$ ,  $\{1,2,3,4\}$ ).

### Basic cycles in permutographs PG with tree-contexture

In this chapter we regard permutographs with a tree-contexture  $CT = T(n)$ .  $T(n)$  is a tree on  $n$  knots (values). The permutograph  $PG(II, T(n))$  represents a network for  $n > 2$ , i.e. the net of negations of a  $n$ -valued logic. This net can be constructed from meshes (Maschen) of minimal length. These constituting meshes are called *basic cycles* (Basiskreise), i.e. cycles without chords. Basic cycles always have the length of 4 or 6, if the contexture of values forms a tree.

**THEOREM 1:** Every tree-contexture  $T(n)$  (tree of negators) produces a permutograph PG containing  $\binom{n-1}{2}$  different basic cycles.

Proof. We label the  $n-1$  edges of the tree of negators by  $N_1, N_2, \dots, N_{n-1}$ . We differentiate neighboured und unneighboured edges of the tree-contexture  $T(n)$ .

In logical terms we express identity as follows:

$$N_i N_j \dots N_k (\pi) = \pi.$$

That means identity corresponds with a cycle in PG. In case of alternating indices of negators we get the identities after a sequence of 4 or 6 negators. Consequently we gain a basic cycle of length 6, if the edges of the tree-contexture  $T$  are neighboured and a basic cycle of length 4, if these edges are unneighboured.

$$\text{6-cycle: } N_i N_j N_i N_j N_i N_j \equiv I \quad (N_i, N_j \text{ neighboured})$$

$$\text{4-cycle: } N_i N_k N_i N_k \equiv I \quad (N_i, N_k \text{ unneighboured})$$

By matching all possible indices — neighboured or not — we get the total number of basic cycles  $\binom{n-1}{2}$ , because every graph, constituting a tree has  $n-1$  edges. ■

There exist permutographs PG with basic cycles only of length 6. The corresponding tree  $T$  of these PG has the shape of a *star*, i.e. one knot of  $T$  is connected with all other  $(n-1)$  knots. Following theorem 1 there are  $\binom{n-1}{2}$  different basic cycles of length 6.

If the tree-contexture  $T$  of PG forms a *line*

$$\square \square \square \dots \square \square$$

we have  $\binom{n-2}{2}$  basic cycles of length 4 and  $n-2$  basic cycles of length 6; in total  $\binom{n-2}{2} + n-2 = \binom{n-1}{2}$  basic cycles.

**THEOREM 2.** Be  $BC4(n)$  respectively  $BC6(n)$  the total number of basic cycles of length 4 respectively of length 6,  $d_i$  the number of knots with degree  $i$  of the tree-contexture  $T(n)$  of PG, then

$$BC6(n) = \sum_{i=2}^r \binom{i}{2} d_i \quad r := \text{maximal degree of a knot in } T(n) \quad 2 \geq r \geq n-1$$

and

$$BC4(n) = \binom{n-1}{2} - BC6(n).$$

Proof: Basic cycles of length 6 corresponds with neighboured edges in  $T(n)$ . A knot of degree  $i$  ( $i > 1$ ) has exactly  $i$  neighbours in  $T(n)$ . Each pair of neighboured edges constitutes a basic cycle of length 6

$$N_i N_j N_i N_j N_i N_j$$

The number  $d_i$  of the matched neighboured edges of a knot with degree  $i$  is  $\binom{i}{2}$ . A knot of degree 1 has only one neighbour so that a matching of neighbours is impossible. It follows the number of

$$BC6(n) = \sum_{i=2}^n \binom{i}{2} d_i$$

basic cycles of length 6. Following theorem 1 there exists  $\binom{n-1}{2} \cdot BC6(n)$  basic cycles, so that

$$BC4(n) = \binom{n-1}{2} \cdot BC6(n). \quad \blacksquare$$

Example:  $CT := T_2(6)$  (see suppl. tab. basic cycles),  $d_1 = 3$ ,  $d_2 = 2$ ,  $d_3 = 1$ ;  $BC6(T_2(6)) = 3\binom{1}{2} + 2\binom{2}{2} + \binom{3}{2} = 0+2+3=5$

**COROLLARY 1:** The total number of basic cycles of length 4 respectively length 6 be  $M4(n)$  resp.  $M6(n)$ . If the value-contexture forms a tree, then

$$M4(n) = \frac{P(r,n) \cdot BC4(n)}{4} \quad M6(n) = \frac{P(r,n) \cdot BC6(n)}{6}$$

**Proof:**  $PG(II,T)$  contains  $P(r,n)$  knots. Every knot  $k$  belongs to the  $BC4(n)$  basic cycles of length 4 and to the  $BC6(n)$  basic cycles of length 6. A basic cycle consists of 4 respectively 6 knots. Therefrom the result follows immediately.  $\blacksquare$

**Remark** (quoted from *Fiorini/Wilson* [3]): In 1973 *Szekeress* introduced two-coloured cycles (this corresponds with the alternating indices) of cubic graphs and called it *basic circuits*. Each edge of  $G$  lies exactly on two of these basic circuits. That also holds for all edges of  $PG$  and  $PG$  must not be cubic.

#### Compositions and Decompositions of Permutographs by Unions or Subgraphs of Tree-contextures

Here are given only some small examples for the understanding of the effect of contextures. A more detailed analysis will appear later. The indices on edges of a permutograph are negator-indices. Integers in squares are names of values. Integers surrounded by cycles are ordinal numbers of permutations in lexicographic order.

##### 1. Decompositions

Because the tree-contextures are minimal contextures to generate a connected permutograph on all  $P(r,n)$  permutations, it is clear, that a contexture, which is a subgraph of such a tree-contexture leads to a decomposition of the  $P(r,n)$ -permutograph.

Example 1:

$$PG_1(4!; \square-\square-\square-\square)$$

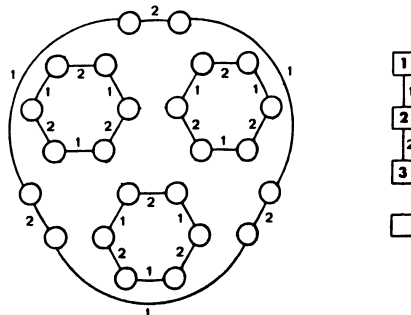
$$CT_1 := \boxed{1} \xrightarrow{N1} \boxed{2} \xrightarrow{N2} \boxed{3} \xrightarrow{N3} \boxed{4}$$

f.e. is a sub-contexture of  $CT_1$

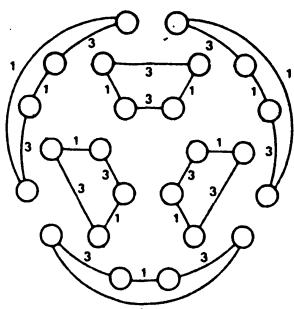
$$CT_{1a} := \boxed{1} \xrightarrow{N1} \boxed{2} \xrightarrow{N2} \boxed{3}$$

$$PG_{1a} := PG(4!; \square-\square-\square) \quad \text{is a decomposition of } PG_1 \text{ into 4}$$

permutographs  $PG(3!; \square-\square-\square)$ .







If you choose the subcontexture

sub CT<sub>1b</sub> :=  $\boxed{1} \boxed{1} \boxed{2} \quad \boxed{3} \boxed{3} \boxed{4}$

then you get a decomposition of PG<sub>1</sub> with 6 components, which are not permutographs (with a tree-contexture) of lower order; but the components have the shape of basic cycles

C<sub>4</sub> := (1313).

Example 2:

PG<sub>2</sub>( 5! ;  $\square-\square-\square-\square-\square$  )

sub CT<sub>2a</sub> :=  $\boxed{1} \boxed{1} \boxed{2} \quad \boxed{4} \boxed{4} \boxed{5}$

PG<sub>2</sub> is decomposed by CT<sub>2a</sub> into 30 components of the shape

C<sub>4</sub> := (1414).

2. Compositions

The union of at least two contextures is called a (contexture-) *composition*.

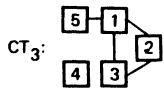
Example 3: Given PG<sub>2</sub> := PG( 5! ; T<sub>1</sub>(5) ) ; CT<sub>2</sub> := T<sub>1</sub>(5) :=  $\boxed{1}-\boxed{2}-\boxed{3}-\boxed{4}-\boxed{5}$  .

A composition of two subgraphs of T<sub>1</sub>(5) – for instance

$\boxed{1}-\boxed{2}-\boxed{3}$  and  $\boxed{3}-\boxed{4}-\boxed{5}$  leads to the complete contexture of PG<sub>2</sub>.

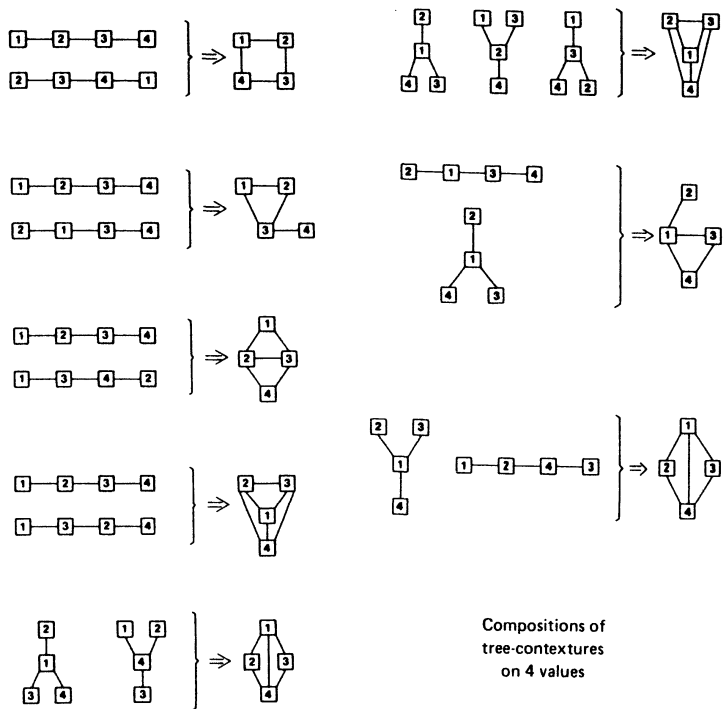
A permutograph PG is called	contexture CT
<i>unbalanced</i>	CT is disconnected
<i>balanced</i>	CT is a tree (a minimal connected graph).
<i>overbalanced</i>	CT is a graph with cycles.

Example 4: CT<sub>3a</sub> :=  $\boxed{1}-\boxed{2}-\boxed{3}$  CT<sub>3b</sub> :=  $\boxed{1}-\boxed{3}-\boxed{5}$  . The union CT<sub>3a</sub> ∪ CT<sub>3b</sub> is

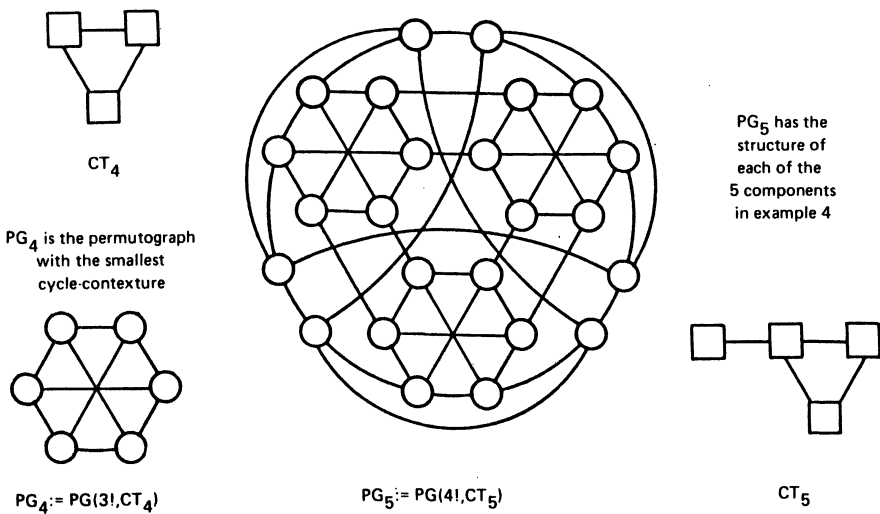


PG<sub>3</sub>( 5! ; CT<sub>3</sub> ) is an unbalanced permutograph with 5 components; each on different 24 permutations (5 x 24 = 120). Each component is an overbalanced permutograph on 4 values (values 1 - 5 except value 4): PG( 4! ; CT<sub>3</sub>/4 ) .

The following contextures are possible compositions of the two tree-contextures for 4 values. It depends on the connections of values – not only on the (tree-) structure of the contextures, if you get a composed contexture with 4,5 or 6 different negations. Two line-contextures are sufficient for a *complete* contexture (i.e. all values are connected with all other values ). By the unions of only star-contextures you need for the complete contexture 3 star-contextures. Of course are all such compositions overbalanced.



The following two permutographs are overbalanced and compositions of minimal permutographs, which are shown above. Permutograph  $PG_4$  has only basic cycles of length 4. This is a consequence of neighbourhood of the values in the cycle-contexture.



### Homogeneity of knots

By construction the permutographs have a certain equality in structure. They are very symmetrical. All knots of a permutograph, e.g. the permutations  $\pi \in \Pi$ , are of equal rank. Each knot is embedded in the same set of cycles without chords\* (basic cycles).

A graph  $G$  is called *knot-homogeneous*, if each knot is embedded in the same (isomorph) set of cycles without chords. Obviously all knot-homogeneous graphs are regular. Knot-homogeneous are for example all cycle-graphs  $C_n$ , complete graphs  $K_n$ , the graphs of the 5 platonic solids (tetrahedron, cube, octahedron, dodecahedron, icosahedron) and also the Petersen graph.

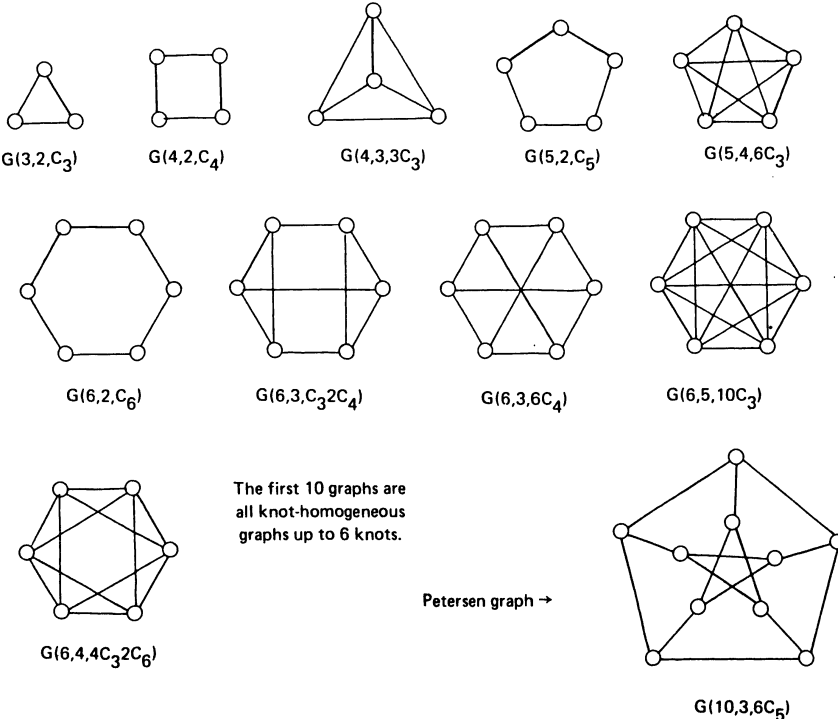
We characterize knot-homogeneous graphs by

$$G_{\text{hom}} := G(n, r, M);$$

$n$  is the number of knots,  $r$  the degree of regularity, and  $M$  is the set of all cycles without chords through some knot. If all cycles  $c \in M$  have the same length, then you get a *perfect symmetry*.

The knowledge of  $M$  is not only useful for the discovering of symmetry or antisymmetry but also helpful for regarding of coverings, decompositions, matroids etc.. Because there is a connection between basic cycles and contextures, it is possible to study the above marked hints from a new point of view.

Be  $M_i$  the set of all cycles  $c$  through a certain knot  $k_i$  of a knot-homogeneous graph  $G_{\text{hom}}$ . From the property of knot-homogeneity follows: all  $M_i$  are isomorph. Also trivial is, that it is always possible to get a  $B \subset M$  so, that  $B$  is sufficient for a complete covering of  $G_{\text{hom}}$  (basic set  $B$ ). Some examples may illustrate the preceding remarks.



\*Edges, which connect knots of a cycle, but do not belong to the cycle are called *chords*.

Supplement

The first two tables of the two balanced permutographs with a line-contexture resp. a star-contexture you can use to work with negations in a 4-valued negation-system. For example holds

in the negations-system with the line-contexture

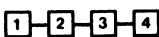
$$\begin{aligned} &N_1N_2N_1N_3N_2N_1 \text{ (1234)} \\ &= N_1N_2N_1N_3N_2N_1 \text{ (1)} \\ &= N_1N_2N_1N_3N_2 \text{ (7)} \\ &= N_1N_2N_1N_3 \text{ (13)} \\ &= N_1N_2N_1 \text{ (19)} \\ &= N_1N_2 \text{ (21)} \\ &= N_1 \text{ (23)} = \text{(24)} = (4321) \end{aligned}$$

in the negation-system with the star-contexture

$$\begin{aligned} &N_1N_2N_1N_3N_2N_1 \text{ (1234)} \\ &= N_1N_2N_1N_3N_2N_1 \text{ (1)} \\ &= N_1N_2N_1N_3N_2 \text{ (7)} \\ &= N_1N_2N_1N_3 \text{ (9)} \\ &= N_1N_2N_1 \text{ (10)} \\ &= N_1N_2 \text{ (4)} \\ &= N_1 \text{ (14)} = \text{(16)} = (3241) \end{aligned}$$

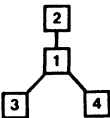
permutation	N1	N2	N3
1234	1	7	3
1243	2	8	4
1324	3	9	1
1342	4	10	2
1423	5	11	6
1432	6	12	5
2134	7	1	13
2143	8	2	14
2314	9	3	15
2341	10	4	16
2413	11	5	17
2431	12	6	18
3124	13	15	7
3142	14	16	8
3214	15	13	9
3241	16	14	10
3412	17	18	11
3421	18	17	12
4123	19	21	20
4132	20	22	19
4213	21	19	23
4231	22	20	24
4312	23	24	21
4321	24	23	22

Table of negations  
Permutograph with line contexture



permutation	N1	N2	N3
1234	1	7	15
1243	2	8	16
1324	3	9	13
1342	4	10	14
1423	5	11	18
1432	6	12	17
2134	7	1	9
2143	8	2	10
2314	9	3	7
2341	10	4	8
2413	11	5	12
2431	12	6	11
3124	13	15	3
3142	14	16	4
3214	15	13	1
3241	16	14	2
3412	17	18	6
3421	18	17	5
4123	19	21	24
4132	20	22	23
4213	21	19	22
4231	22	20	21
4312	23	24	20
4321	24	23	19

Table of negations  
Permutograph with star contexture










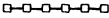











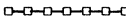
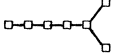
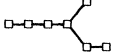

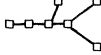






n	PG generating tree-contexture T(n)	Number of different basic cycles			Basic cycles (indices of negators)	Total number of basic cycles in PG		
		■	●	Σ		■	●	Σ
2	 $T_1(2)$							
3	 $T_1(3)$	1		1	121212		1	1
4	 $T_1(4)$	1	2	3	1313 121212 232323	6	8	14
	 $T_2(4)$	-	3	3	121212 131313 232323	-	12	12
5	 $T_1(5)$	3	3	6	1313 1414 2424 121212 232323 343434	90	60	150
	 $T_2(5)$	2	4	6	1313 1414 242424 343434 121212 232323	60	80	140
	 $T_3(5)$	-	6	6	121212 131313 141414 232323 242424 343434	-	120	120
6	 $T_1(6)$	6	4	10	1313 1414 1515 2424 2525 3535 121212 232323 343434 454545	1080	480	1560
	 $T_2(6)$	5	5	10	1313 1414 1515 2424 2525 353535 454545 121212 232323 242424	900	600	1500
	 $T_3(6)$	3	7	10	1313 1414 1515 252525 343434 353535 454545	540	840	1380
	 $T_4(6)$	4	6	10	121212 1414 1515 2424 2525 343434 353535 454545	720	720	1440

Table: Basic cycles in permutographs (with tree-contextures) on n-permutations

n	PG generating tree-contexture T(n)	Number of different basic cycles			Basic cycles (indices of negators)		Total number of basic cycles in PG		
				$\Sigma$					$\Sigma$
continue 6	 $T_5(6)$	5	5	10	1414 1515 2525 3434 4545	121212 131313 232323 242424 353535	900	600	1500
	 $T_6(6)$	—	10	10	121212 131313 141414 151515 232323 242424 252525 343434 353535 454545				
7	 $T_1(7)$	10	5	15	In T(n) unneighbourbed edges form 4-cycles In T(n) neighbourbed edges form 6-cycles		12600	4200	16800
	 $T_2(7)$	9	6	15			11340	5040	16380
	 $T_3(7)$	9	6	15			11340	5040	16380
	 $T_4(7)$	9	6	15			11340	5040	16380
	 $T_5(7)$	8	7	15			10080	5880	15960
	 $T_6(7)$	8	7	15			10080	5880	15960
	 $T_7(7)$	7	8	15			8820	6720	15540
	 $T_8(7)$	7	8	15			8820	6720	15540
	 $T_9(7)$	6	9	15			7560	7560	15120
	 $T_{10}(7)$	4	11	15			5040	9240	14120
	 $T_{11}(7)$	—	15	15			—	12600	12600

**Acknowledgement**

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