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A SURVEY OF BOUNDARY VALUE PROBLEMS FOR BUNDLES OVER COMPLEX SPACES

ADAM HARRIS

ABSTRACT. This article is the expanded version of a conference paper presented at the 21st Winter School on Geometry and Physics, Srni, Czech Republic, January 14th – 20th, 2001. The following is a summary of recent results of the author concerning solutions of the Cauchy-Riemann equation and Laplace equation around an isolated complex singularity, with applications to the problem of analytic continuation of Hermitian-holomorphic vector bundles.

1. THE CAUCHY-RIEMANN EQUATION ON A PUNCTURED NEIGHBOURHOOD

Let X be a reduced n-dimensional complex space, for which the set of singularities consists of finitely many points. If $X' \subseteq X$ denotes the set of smooth points, we will consider a holomorphic vector bundle $E \to X' \setminus A$, where A represents a closed analytic subset of complex codimension at least two. It will be assumed moreover that E comes equipped with a Hermitian metric h. The results surveyed here will provide criteria for holomorphic extension of E across A, or across the singular points of X if $A = \emptyset$. A similar survey was carried out in [6] assuming the existence of certain special connections on E, and assuming X to be everywhere smooth. The more fundamental approach taken here is via the metric h, and in particular via the L^2 -theory of the Cauchy-Riemann equation for differential (p,q)-forms with coefficients in E (cf. e.g., Demailly [4]).

We begin with a brief review of the $\bar{\partial}$ -Neumann problem for strongly pseudoconvex domains. Let $\mathcal{H}_1 \stackrel{S}{\to} \mathcal{H}_2 \stackrel{T}{\to} \mathcal{H}_3$ be a sequence of Hilbert spaces, exact with respect to unbounded linear operators S and T. If S^* and T^* denote the corresponding adjoints, recall that im(S) = ker(T) if there exists a positive constant C for all $x \in Dom(T) \cap Dom(S^*)$ such that

$$||S^*x||_{\mathcal{H}_1}^2 + ||Tx||_{\mathcal{H}_3}^2 \ge C||x||_{\mathcal{H}_2}^2.$$

In particular, consider $\mathcal{H}_q:=L^2(\Omega, \bigwedge^{p,q}(E))$, where Ω is a domain in \mathbb{C}^n with C^2 -boundary, and $\bigwedge^{p,q}(E)$ denotes the forms of type (p,q) with coefficients in a Hermitian-holomorphic vector bundle $E\to\Omega$. S and T consequently represent the Cauchy-Riemann operators $\bar{\partial}_E: \bigwedge^{p,q}(E)\to \bigwedge^{p,q+1}(E)$. In this context \mathcal{H}_q may be regarded as the metric completion of compactly supported forms $C_c^\infty(\Omega, \bigwedge^{p,q}(E))$ with

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respect to the L^2 -inner product induced by h. While the formal adjoint of $\bar{\partial}_E$ on compactly supported forms is simply defined via integration by parts, the appearance of a boundary term obstructs the definition of $\bar{\partial}_E^*$ when applying Stokes' theorem over \mathcal{H}_q . Specifically $\alpha \in L^2(\Omega, \Lambda^{p,q-1}(E))$, $\beta \in L^2(\Omega, \Lambda^{p,q}(E))$ implies

$$(\bar{\partial}\alpha,\beta) = (\alpha,\bar{\partial}^*\beta) + \int_{\partial\Omega} \langle \nu^{0.1} \wedge \alpha,\beta \rangle$$

or equivalently $\alpha \in L^2(\Omega, \bigwedge^{p,q+1}(E))$ implies

$$(\bar{\partial}^*\alpha,\beta) = (\alpha,\bar{\partial}\beta) + \int_{\partial\Omega} \langle \nu^{0.1} \vee \alpha,\beta \rangle,$$

where $\nu^{0.1}$ denotes the (0.1)-component of the unit normal covector to $\partial\Omega$, and \vee denotes the adjoint of exterior multiplication with respect to the pointwise inner product on forms. We therefore restrict the domain of definition of $\bar{\partial}_E^*$ to those forms α such that

$$\nu^{0.1} \vee \alpha \mid_{\partial\Omega} = \nu^{0.1} \vee \bar{\partial} \alpha \mid_{\partial\Omega} = 0$$
.

We now introduce the self-adjoint "Laplace-Beltrami" operator $\Box = \bar{\partial}_E \bar{\partial}_E^* + \bar{\partial}_E^* \bar{\partial}_E$. Given $\eta \in C^{\infty}(\Omega, \bigwedge^{p,q}(E)) \cap C^0(\overline{\Omega}, \bigwedge^{p,q}(E))$, both existence and smoothness of a solution to the equation $\Box \alpha = \eta$ are then guaranteed by a stronger estimate than (*), of the form

$$(\neg \varphi, \varphi) \ge C \|\varphi\|^2 + \int_{\partial\Omega} |\varphi|^2 + \|\nabla^{0.1}\varphi\|^2$$

for all φ in the restricted domain, where

$$\nabla^{0.1}: \bigwedge^{p,q}(E) \to (T^{0.1}X)^* \otimes \bigwedge^{p,q}(E)$$

denotes the natural connection on forms induced by complex conjugation of the Chern connection of E. The positive constant C was moreover shown by Hörmander to exist for all such φ when the boundary $\partial\Omega$ is strongly pseudoconvex [9]. With this fact we are now able to solve the " $\bar{\partial}$ -Neumann problem" using the theory developed for this purpose by Kohn and Spencer [10]. Namely, if η is smoothly defined on a strongly pseudoconvex domain Ω then there exists

$$(\dagger) \qquad \qquad \varphi \in \{\alpha \in C^{\infty}_{L^{2}}(\Omega, \bigwedge^{p,q}(E)) \mid \nu^{0.1} \vee \alpha \mid_{\partial \Omega} = \nu^{0.1} \vee \bar{\partial} \alpha \mid_{\partial \Omega} = 0\}$$

such that $\Box \varphi = \eta$. Using the additional assumption that $\bar{\partial} \eta = 0$, we obtain an immediate solution of the Cauchy-Riemann equation $\bar{\partial} u = \eta$, since

$$0 = (\bar{\partial}\eta, \bar{\partial}\varphi) = (\bar{\partial}\bar{\partial}^*\bar{\partial}\eta, \bar{\partial}\varphi) = \|\bar{\partial}^*\bar{\partial}\varphi\|^2$$

(given the condition $\nu^{0.1} \vee \bar{\partial} \varphi \mid_{\partial \Omega} = 0$), and hence $u = \bar{\partial}^* \varphi$.

While the unit ball $B \subset \mathbb{C}^n$ is the prototype for all strictly pseudoconvex domains, it is perhaps surprising that almost nothing was known about solvability of the Cauchy-Riemann equation on the punctured ball $B \setminus \{0\}$, prior to the appearance of a paper of S. Bando in 1991 [1]. Consider a Hermitian-holomorphic vector bundle $(E, h) \to B \setminus \{0\} \subset \mathbb{C}^2$ such that the curvature form F_h , represented in any holomorphic frame of E by the (1,1)-form $\bar{\partial}(h^{-1}\partial h)$, belongs to $L^2(B \setminus \{0\})$. For η a $\bar{\partial}$ -closed, compactly supported (0,1)-form on $B \setminus \{0\}$, Bando solved the equation $\bar{\partial}u = \eta$ in two steps. Given $B_{\epsilon,1} := \{z \in \mathbb{C}^2 \mid 0 < \varepsilon < |z| < 1\}$, it was first shown that for all $\varepsilon > 0$ the equation $\Box \varphi_{\varepsilon} = \eta$ is solvable on $B_{\epsilon,1}$, for φ_{ε} a smooth (0.1)-form satisfying the

 $\bar{\partial}$ -Neumann conditions (†) on |z|=1, and the Dirichlet condition $\varphi_{\varepsilon}=0$ on $|z|=\varepsilon$. The key idea of this part of Bando's argument is to manipulate Weitzenböck formulae expressing the Laplace-Beltrami operator in terms of the curvature form F_h , which is assumed to belong to $L^2(B\setminus\{0\})$. In this way he derives an estimate of the form (**) independently of ε , and obtains a smooth solution φ on $B\setminus\{0\}$ satisfying $\|\varphi\|\leq \|\eta\|$, taking the uniform limit as ε approaches zero.

The second part of the proof treats the vanishing of $\bar{\partial}^*\bar{\partial}\varphi$ – a much more delicate issue in the case of the punctured ball – for which Bando recalls a Moser iteration technique developed in earlier work with Kasue and Nakajima [2]. Solvability of the equation $\bar{\partial}u = \eta$ such that $||u|| \leq ||\eta||$ is then applied in the standard way to obtain a sufficiently large number N of global holomorphic sections generating E over $B \setminus \{0\}$, and hence an embedding $E \hookrightarrow \mathbb{C}^N \times B \setminus \{0\}$. It follows from Hartogs' theorem that the holomorphic structure of E extends uniquely across the origin as a coherent analytic sheaf, though this structure may only be assumed to be locally free when $B \setminus \{0\} \subset \mathbb{C}^2$.

In collaboration with Y. Tonegawa [7], the author sought to generalise Bando's removable singularities theorem by first solving the Cauchy-Riemann equation on $B \setminus \{0\} \subset \mathbb{C}^n$. With an appropriate adjustment of the Hölder and Sobolev exponents it is a relatively straightforward matter to apply Bando's analysis of the $\bar{\partial}$ -Neumann/Dirichlet problem to solve the Cauchy-Riemann equation in higher dimension when the curvature F_h belongs to $L^n(B \setminus \{0\})$. In fact, the closed (0.1)-form η need not be compactly supported, but must also belong to $L^n(B \setminus \{0\})$ (cf. [7]). A possibly surprising observation in the case n=2 is the consequent vanishing of the L^2 Dolbeault cohomology $H_{L^2}^{0,1}(B \setminus \{0\})$, as compared with the infinite dimensionality of $H^{0,1}(B \setminus \{0\})$.

2. Removable singularities for holomorphic vector bundles

Given $(E,h) \to B \setminus \{0\} \subset \mathbb{C}^n$, recall that the equation $\bar{\partial}u = F_h$ determines the cohomology obstruction to existence of a holomorphic connection on E (cf [3]). When F_h belongs to $L^n(B \setminus \{0\})$, solvability of the $\bar{\partial}$ -Neumann/Dirichlet problem, together with previous work of N.P. Buchdahl and the author [3], consequently shows that E admits a unique locally free extension across the origin. For the more general case of a complex manifold X, containing an analytic subset A of complex codimension at least two, with Hermitian-holomorphic vector bundle $(E,h) \to X \setminus A$, a unique holomorphic extension of E across E is carried out by first stratifying E into smooth open subsets. At any point of this stratification, the restriction of E to a normal slice corresponds to the extension problem for the punctured ball. The main theorem of [7] may be stated as follows.

Theorem 1. Let $(E,h) \to X \setminus A$ be a Hermitian-holomorphic vector bundle such that $F_h \in L^n(X \setminus A)$, then there exists a unique vector bundle $\hat{E} \to X$ such that $\hat{E} \mid_{X \setminus A} \cong E$.

A priori, E will have L^n -curvature when restricted to almost every normal slice. The uniform extension of E across A therefore requires two fundamental lemmas.

Lemma 1. (cf., e.g., [13, Lemma 3.1], also [14, Proposition 6.8].) Let Ω be a domain in \mathbb{C}^{n-1} , Δ a disc in \mathbb{C} , and $\Delta^* := \Delta \cap \mathbb{P}_1 \setminus \{0\}$. Given $g \in \mathcal{O}(\Omega \times \Delta^*, \mathbb{GL}(r, \mathbb{C}))$, suppose there exists $\omega \in \Omega$, $h^+ \in \mathcal{O}(\Delta, \mathbb{GL}(r, \mathbb{C}))$, and $h^- \in \mathcal{O}(\mathbb{P}_1 \setminus \{0\}, \mathbb{GL}(r, \mathbb{C}))$ such that

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 $g(\omega,...)=h^+\cdot h^-$. Then there exists a uniquely defined analytic hypersurface $\Gamma\subset\Omega$, and unique holomorphic matrix-valued functions $g^+\in\mathcal{O}((\Omega\setminus\Gamma)\times\Delta,\mathbb{GL}(r,\mathbb{C})),\ g^-\in\mathcal{O}((\Omega\setminus\Gamma)\times\mathbb{P}_1\setminus\{0\},\mathbb{GL}(r,\mathbb{C}))$ such that

(i) $g = g^+ \cdot g^-$,

(ii) $g^{-}(...,\infty) \equiv 1$,

(iii) g^+, g^- extend to meromorphic matrix-valued functions on $\Omega \times \Delta$ (resp. $\Omega \times \mathbb{P}_1 \setminus \{0\}$) with polar locus $\Gamma \times \Delta$ (resp. $\Gamma \times \mathbb{P}_1 \setminus \{0\}$).

Lemma 2. (Rothstein, cf. [13], Theorem 1.12.) Let Δ^{n-1} be a polydisc in \mathbb{C}^{n-1} , and $\Delta(r)$ a disc of radius r. Suppose f is a meromorphic function on $\Delta^{n-1} \times \Delta(r)$, such that $f_w := f(w, ...)$ extends meromorphically to $\Delta(r + \varepsilon)$ for all $w \in \Delta^{n-1}$. Then f extends meromorphically to $\Delta^{n-1} \times \Delta(r + \varepsilon)$.

The proof of extension via slicing follows from a straightforward induction, which is a simplified version of an argument used by Shevchishin for his "Thullen-type" extension theorem [13]. Due to an additional slicing technique of Siu [14], the statement of the main theorem may be generalised to include any closed set A of finite 2n-4-dimensional Hausdorff measure. We recall moreover that sharpness of the main hypothesis follows from the example of $E \to \mathbb{C}^n \setminus \{0\}$ corresponding to the pullback of the holomorphic tangent bundle from \mathbb{CP}_{n-1} , $n \geq 3$. It is easily seen that E admits no locally free holomorphic extension across the origin (cf. [3]). For h corresponding to the pullback of the Fubini-Study metric, however, it can be seen by direct computation that $F_h \in L^p(B \setminus \{0\})$ for all p < n.

Now consider X a reduced and irreducible n-dimensional complex space with isolated singularity $x_0 \in X$. Let $\rho: X \to [0, \infty)$, $\rho(x_0) = 0$, be a strongly plurisubharmonic function, and $\Omega \subset X$ a neighbourhood of x_0 with smooth compact boundary $\Sigma = \{x \in X \mid \rho(x) = c < \infty\}$. It will be assumed $\Omega_0 := \Omega \setminus \{0\}$ is a complex manifold with Kähler form $\omega = i\bar{\partial}\partial\rho$ and associated metric g satisfying the following conditions:

where R_g denotes the canonical curvature form associated with g. It will further be assumed that the Sobolev inequality holds with respect to this metric, i.e.,

(ii)
$$\left(\int_{\Omega_0} |f|^{\frac{2n}{n-1}} \omega^n \right)^{\frac{n-1}{n}} \le c(n) \int_{\Omega_0} |\nabla f|^2 \omega^n$$

for smooth compactly supported functions f. In addition, (iii) let $\delta(x_0, x)$ denote the Riemannian metric distance function on Ω_0 , and let $B_{\delta}(x_0, r)$ be the associated ball of radius r. For some sufficiently small 0 < c' < c it will be assumed that there exists a positive constant K such that

$$\int_{B_{\delta}(x_0,r)} \omega^n \le Kr^{2n}, \quad \text{for all} \quad 0 < r \le c'.$$

In fact, when $X \subset \mathbb{C}^m$ is an affine analytic variety, with ρ corresponding to the restriction of the Euclidean norm-squared function, conditions (ii) and (iii) hold automatically. Moreover, the curvature form $R_g = \beta \wedge \beta^*$, where β denotes the second fundamental form of the embedded variety, and it is an easy computation to show that

$$\int_{\Omega_0} |R_g|^n \le \int_{\Omega_0} |\beta|^{2n} < \infty$$

when, for example, $X: z_{n+1}^k = f(z_1, ..., z_n)$ is an analytic hypersurface in \mathbb{C}^{n+1} with isolated singularity at the origin, such that $2 \leq k < ord_f(0)$. On the other hand surfaces in \mathbb{C}^3 defined by an equation of the form $z^k = xy$, which constitute a special class of orbifold singularities (cf., e.g., [5]), do not satisfy condition (i) with respect to the restricted ambient metric. For any singular space X of this type, corresponding to the quotient of \mathbb{C}^n by a finite subgroup of $\mathbb{SU}(n)$, the most natural choice of ρ is that induced by $|z|^2$ on the Euclidean covering space, since the associated orbifold metric is flat.

Let $(E,h) \to X_0$ be a Hermitian-holomorphic vector bundle, with Kähler metric g on X_0 derived from $i\bar{\partial}\partial\rho$ as above, so that the curvature F_h belongs to $L^n(\Omega_0)$. If $\eta \in C^\infty(X_0, \bigwedge^{0.1}(E))$ is a $\bar{\partial}$ -closed (0.1)-form also belonging to $L^n(\Omega_0)$, then it is shown in [8], theorem 1, that the equation $\bar{\partial}u = \eta$ admits a smooth solution on Ω_0 such that $\|u\| \leq \|\eta\|$. Once again the proof is an appropriate modification of the method of Bando in solving the $\bar{\partial}$ -Neumann/Dirichlet problem on $\Omega \setminus B_\delta(x_0, \varepsilon)$, before taking the limit as ε approaches zero. In obtaining the basic estimate (**) for existence and regularity of solutions, the curvature of h and g must be taken into account from the Weitzenböck formula

$$\begin{split} (\Box \varphi, \varphi) &= \|\bar{\partial}_E \varphi\|^2 + \|\bar{\partial}_E^* \varphi\|^2 = \|\nabla^{0.1} \varphi\|^2 + \int_{\Sigma} |\varphi|^2 + \\ \varphi_{\bar{a}}^{\alpha} g^{b\bar{a}} R_{bk\bar{l}}^k h_{\alpha\bar{\beta}} \varphi_{\bar{l}}^{\beta} + \varphi_{\bar{i}}^{\alpha} g^{k\bar{i}} R_{bk\bar{l}}^b h_{\alpha\bar{\beta}} \varphi_{\bar{l}}^{\beta} + \varphi_{\bar{i}}^{\gamma} g^{k\bar{i}} F_{\gamma k\bar{l}}^{\alpha} h_{\gamma\bar{\zeta}} \varphi_{\bar{l}}^{\zeta} \end{split}$$

Now by means of a standard application of L^2 -theory of the Cauchy-Riemann equation, it is once more possible to embed $E\mid_{\Omega_0}$ in the trivial bundle $\mathbb{C}^N\times\Omega_0$, for N sufficiently large. We now obtain [8]

Theorem 2. Let X^n be a reduced complex space with normal isolated singularity at $x_0 \in X$, and $\rho: X \to [0, \infty)$ a smooth, strongly plurisubharmonic exhaustion function centred at x_0 which satisfies the conditions (i) – (iii) above. If $E \to X_0$ is a Hermitian-holomorphic vector bundle with L^n_{loc} -curvature, then there exists a reflexive sheaf $\mathcal{F} \to X$ such that $\mathcal{F}|_{X_0} \cong \mathcal{O}(E)$.

A further consequence of the solubility of the $\bar{\partial}$ -Neumann problem on Ω_0 is the solvability of the equation $\bar{\partial}u=F_h$, which implies existence of a holomorphic connection on $E\to\Omega_0$ (cf. [7]). The extension argument of [3], theorem 2.2 will then automatically imply the following

Corollary 1. Let X^n be a reduced analytic space with isolated singularity $x_0 \in X$ and strongly plurisubharmonic function $\rho: X \to [0, \infty)$ satisfying conditions (i)-(iii). Consider $\pi: Y \to X$ to be a surjective holomorphic map from a complex manifold Y such that $\pi^{-1}(x_0)$ has codimension greater than one. If $E \to X_0$ is a Hermitian-holomorphic vector bundle with L^n_{loc} -curvature, then there exists a unique holomorphic vector bundle $V \to Y$ such that $V \mid_{Y \setminus \pi^{-1}(x_0)} \cong \pi^* E$.

A natural instance of this result occurs when π corresponds to a quotient of \mathbb{C}^n under the action of a finite group $G \subset \mathbb{SU}(n)$, i.e., X has an orbifold singularity at x_0 . Another potential class of examples corresponds to isolated singularities with "small resolution". Explicit examples of such singularities, with X a hypersurface in \mathbb{C}^4 and $\pi^{-1}(x_0) \cong \mathbb{CP}_1$ were presented in [12]. At present an example which admits a strongly plurisubharmonic function ρ of the required type is not known to the author, however.

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Another application concerns the problem of holomorphic extension from the strictly pseudoconvex CR-hypersurface Σ corresponding to $\rho = c$. Let $\sigma : \Sigma \to E$ be a section of $E \to X_0$, such that σ is closed with respect to the tangential Cauchy-Riemann operator on $E \mid_{\Sigma}$, i.e., $\bar{\partial}_b \sigma = 0$ (cf. [10]). Let $s: \Omega_0 \to E$ be a smooth extension of σ , with support in an arbitrarily small neighbourhood of Σ . From the standard theory of the tangential Cauchy-Riemann complex for Σ , we note that $\bar{\partial}_b \sigma = 0$ if and only if the (n, n-1)-form $\xi = \bar{\partial}^* \star \bar{s}$ satisfies the $\bar{\partial}$ - Neumann conditions (†), where \star denotes the Hodge duality operator on forms ([10], Proposition 5.2.2). The essential idea of the extension technique of Kohn and Rossi [16] is to obtain a solution to the equation $\hat{\partial}^* u = \xi$ such that u again satisfies the conditions (†), and this is done in a manner entirely analogous to the theory of the equation $\bar{\partial}u = \xi$ for a (0,1)-form ξ . Moreover, u satisfies $(a)\bar{\partial}(s-\star\bar{u})=0$ and $(b)\star\bar{u}\mid_{\Sigma}=0$, hence $s-\star\bar{u}$ is a holomorphic extension of σ . For the case of Ω_0 corresponding to the punctured neighbourhood of an isolated singularity our adaptation of Bando's method is applied to this end. The argument here also is essentially a dualised version of the method of the previous sections, and goes through with only minor alterations.

Corollary 2. Let X^n be a reduced analytic space with isolated singularity x_0 , and let $\rho: X \to [0, \infty)$ be a strongly plurisubharmonic function satisfying the conditions (i)–(iii). If $E \to X_0$ is a Hermitian-holomorphic vector bundle with L^n_{loc} -curvature, and σ a $\bar{\partial}_b$ -closed section of $E \mid_{\Sigma}$, then there exists a unique holomorphic extension of σ as a section of E on Ω_0 , and hence as a section of the reflexive sheaf F on Ω .

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